Preface

I have written this book for students who have had one or two years of calculus and little else. The most important prerequisite is the realization of the need to use mathematics in their studies or work. I should say at the outset that this is not a mathematics book in the sense that I do not prove many theorems and may have occasional lapses of the degree of rigor that would satisfy a pure mathematician. However, I have endeavored to present theorems that we use in a manner that is both accurate and intelligible. (I don't know who first said it, but there is a saying that "Pure mathematicians don't trust applied mathematicians and applied mathematicians don't understand pure mathematicians.") There are entire books that discuss each of the topics that we discuss in a single chapter or even less, so we can give only an introduction to each. I have tried to make my treatment of each topic self-contained, but I would consider it a great success if you became interested enough in any topic that you sought out further study by going to a more detailed treatment (not because my treatment is opaque, but because you want to know more!). The references at the end of each chapter should get you started in this direction.

The first chapter is a review of calculus, which some readers may find easy or superfluous, while others may find it to be helpful. Its purpose is to bring every one up to speed and to provide practice for those whose math is rusty. Although the treatment is elementary, I introduce the ϵ-δ notation for the definition of limits and continuity and then discuss the idea of uniform continuity and uniform convergence of integrals. Even though your interest may be not lie in mathematical rigor, you should at least be aware of when you can interchange limiting operations. For example, when can we write

$$\frac{dF}{dx} = \int_0^\infty \frac{\partial f(x,t)}{\partial x} dt$$

if $F(x) = \int_0^\infty f(x,t)\, dt$? Chapter 2, in which we discuss series, is also a review of material that is treated in all calculus courses. The use and manipulation of series play such an important role throughout applied mathematics that it is important to appreciate the concept of convergence and when certain operations such as term-by-term differentiation and term-by-term integration are valid. In Chapter 3, we introduce a number of non-elementary functions such as the gamma function, the error function, and the Dirac delta function that are defined by certain integral expressions, and then in Chapter 4, we discuss complex numbers, the complex plane, and very briefly, the properties of functions of complex variables. We introduce vectors in Chapter 5 and illustrate the power of vector notation by applying it to a number of problems in analytic geometry that are fairly easy using vector notation but would be tedious without it. Functions of more than one variable are discussed in Chapter 6. This material leads into Chapter 7, where we discuss vector calculus, which is indispensable throughout the physical sciences and engineering. After discussing various coordinate systems in Chapter 8, we go on to discuss linear algebra and vector spaces in Chapter 9 and then matrices and eigenvalue problems in

Chapter 10. The next four chapters constitute a segment on differential equations, including nonlinear differential equations and phase space in Chapter 13 and special functions and Sturm-Liouville theory in Chapter 14. The next two chapters treat Fourier series and their application to solving partial differential equations by the method of separation of variables. We continue our study of partial differential equations in Chapter 17, where we discuss integral transforms, particularly Laplace Transforms and Fourier Transforms. The need to invert Laplace transforms leads naturally to functions of complex variables and integration in the complex plane. Complex variable theory is one of the most profound and beautiful subjects in applied mathematics and all physical science and engineering students should have some familiarity with this subject, even if their work doesn't often require it. In Chapter 19, we show how complex variable theory can be used to evaluate real integrals, to sum series into closed forms, to solve boundary value problems, and to solve fluid-flow problems. The final two chapters discuss probability theory and mathematical statistics. In particular, we discuss confidence intervals, good-of-fit tests, and regression and correlation in the last chapter.

No one can learn this material (nor any thing in the physical sciences nor in engineering, for that matter) without doing lots of problems. For this reason, I have provided at least 15 to 20 problems at the end of each section. These problems range from filling in gaps in the material presented in the section to extending the material, but most often to illustrate applications of the material. All tolled, there are almost 3000 problems in the book. I have provided answers to many of them at the back of the book.

There are a number of powerful commercial computer packages that are available nowadays that can be used to solve many of the problems in this book. These programs not only provide numerical answers, but can also perform algebraic manipulations, and for that reason are called computer algebra systems (CAS). Some of the prominent CAS are MatLab, Maple, Mathematica, and MathCad. I happen to know and use Mathematica, and I have presented examples of one-line Mathematica commands throughout the book that can be used to solve given problems. These commands are just meant to provide examples of the utility of any CAS, and there are a number of problems that ask you to "use any CAS to solve ...." These programs are so available and user-friendly that you might wonder at times "Why do I need to learn all the stuff in this book when I could use a CAS to solve my problem?" I think that no one with any experience would disagree when I say that these programs are a wonderfully useful supplement to the material in this book, but are no substitute for it. During the writing of this book, I found countless examples where a thoughtless use of a CAS would lead you astray. In spite of their friendliness, as in most things, you have to know what you're doing first in order to use them with confidence.

A singular feature of the book is the inclusion of biographies at the beginning of each chapter. Many of the mathematicians that we refer to were rather colorful characters, and I personally find it enjoyable reading about them. I wish to thank my publisher for encouraging me to include them and my wife, Carole, for researching the material for them and for writing every one of them. Each one could easily have been several pages long, and it was difficult to cut them down to one page. We both wish to acknowledge a terrific web-site at the University of St. Andrews in Scotland (www-history.mcs.st-and.ac.uk, and yes, the www- is correct) that lists hundreds of biographies of famous mathematicians, as well as other mathematical subjects.

You read in many prefaces that "this book could not have been written and produced without the help of many people," and it is definitely true. I am particularly grateful to my reviewers, Dennis DeTurck of the University of Pennsylvania, Scott Feller of Wabash College, David Wunsch of the University of Massachusetts at Lowell, Mervin Hanson at Humboldt State University, and Heather Cox at the California Institute of Technology, who slogged through first drafts of all the chapters and who made many great suggestions. I also wish to thank Christine Taylor and her staff at Wilsted & Taylor Publishing Services for coordinating the entire project and correcting all my spelling errors, Bob Ishi for designing his usual beautiful-looking and inviting book, Jane Ellis for dealing with many of the production details and for procuring all the photographs and likenesses for the biographies, Mervin Hanson for rendering over 700 figures in Mathematica and keeping them all straight in spite of countless alterations, John Murdzek for a very helpful copyediting, Paul Anagnostopoulos for composing the entire book, and my publisher Bruce Armbruster and his wife and associate, Kathy, for being the best publishers around and good friends in addition. Finally, I wish to thank my wife, Carole, for preparing the manuscript in TeX, for reading many of the chapters, and for being my best critic, in general.

There are bound to be both typographical and conceptual errors in a book of this breadth and length, and I would appreciate your letting me know about them so that they can be corrected in subsequent printings. I also would welcome general comments, questions, and suggestions either at mquarrie@mcn.org or through the University Science Books website, www.uscibooks.com, where any ancillary material or notices will be posted.