8

MOMENTUM

So far, both in our discussions of Newton's laws and in our use of energy methods, we have been careful to consider interactions between objects that could be described in terms of constant or, at least, uniformly increasing forces acting over some extended time. Nothing ever banged into anything else in a kind of sharp collision. Our reasons for this were good ones. The forces involved in collisions are hard to describe, making the use of Newton's laws difficult. Also, collisions often result in forms of energy (sound, heat, deformation, etc.) that are difficult to account for, making energy calculations impossible. However, colliding objects are very common, and so we need to learn to deal with them. One of the keys to handling collisions is the concept of "impulse."

8.1 Impulse and Momentum

In our study of energy, we discovered that there is a quantity called work that changes an object's mechanical energy. The work is found by multiplying the applied force by the *distance* over which the force acts. In this section, we will find that there is another interesting and useful quantity that comes from multiplying an applied force by the *time* over which it acts.

Let us begin where we are comfortable, with a constant applied force. We then define a quantity called the *impulse* as the product of the force and the time interval over which it acts. There is no conventional symbol for impulse, so we will just write it

$$\overrightarrow{\text{Impulse}} = \vec{F}_{\text{APP}}\Delta t, \tag{8.1}$$

where $\vec{F}_{\text{APP}}$ is the constant applied force and Δt is the time over which it acts. Just as there is no conventional symbol for impulse, there is also no named unit. Impulse is measured, as can be seen from the definition, in newton · seconds, or N · s. Alternatively, since a newton is itself a kg · m/s^2, the impulse can also be expressed as a kg · m/s. Note that impulse is a vector that points in the same direction as the applied force.

So what does this accomplish? Let us go back to Newton's second law and assume that a constant applied force acts on an object that is otherwise in equilibrium. This means that the net force is the applied force, and, if the applied force is constant, then the acceleration will be constant. For constant acceleration, the impulse of the applied force will produce a change in the motion characterized by

$$\begin{aligned}
\overrightarrow{\text{Impulse}} &= \vec{F}_{\text{APP}}\Delta t = \vec{F}_{\text{NET}}\Delta t = m\vec{a}\Delta t \\
&= m\left(\frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}\right)\Delta t \\
\overrightarrow{\text{Impulse}} &= m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}}.
\end{aligned}$$

Thus, an impulse (a force times the time interval over which it acts) causes a change in the quantity $m\vec{v}$. We have actually seen this quantity before. Back on page 81, we saw that this was the way Newton defined his "quantity of motion." We now call this the *momentum*, defined $\vec{p} = m\vec{v}$. This definition allows us to write the effect of an impulse on an object in a form called the impulse–momentum equation:

$$\overrightarrow{\text{Impulse}} = \vec{p}_{\text{f}} - \vec{p}_{\text{i}} \qquad \text{or} \qquad \vec{p}_{\text{i}} + \overrightarrow{\text{Impulse}} = \vec{p}_{\text{f}}. \tag{8.2}$$

The last version serves to emphasize that impulse is the thing that changes an object's momentum from its initial value to its final value. Let us also note that, since impulse and momentum are added together in Equation 8.2, they must have the same units, and indeed they do. Momentum is measured in kg·m/s (or N·s), just like impulse.

We came to Equation 8.2 by assuming a constant force, but we will insist that the impulse of a variable force be defined in such a way that Equation 8.2 always applies.

EXAMPLE 8.1 Momentum of a Bumper Car

Two children are riding in a bumper car at the fair. The mass of the car plus riders is 300 kg. The car is moving straight north at 3 m/s just before it collides head-on with the wall. After the collision, the bumper car is traveling straight south at 2 m/s. What is the impulse that acts on the car during the collision?

ANSWER Because the quantities involved are vectors, we need to be explicit about our coordinate system. Let's choose an axis along the north–south direction and take north to be positive. Then the initial velocity is northward at +3 m/s and the final velocity is –2 m/s north. Equation 8.2 then gives the impulse as

$$\begin{aligned}\overrightarrow{\text{Impulse}} &= \vec{p}_{\text{f}} - \vec{p}_{\text{i}} = m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}} \\ &= 300\text{ kg }(-2\text{ m/s}) - 300\text{ kg }(+3\text{ m/s}) \\ &= -600\text{ kg}\cdot\text{m/s} - 900\text{ kg}\cdot\text{m/s} \\ &= -1500\text{ kg}\cdot\text{m/s}.\end{aligned}$$

The impulse is negative, with positive taken north, so it is toward the south. The wall stopped the northward momentum of the bumper car and then produced a momentum southward.

If you think about the force on the bumper car by the wall during the collision, it must start out small (actually zero until contact is made), grow sharply as the bumper compresses against the wall, peak at some value, and then drop quickly back to zero as the bumper car rebounds. All of this happens in perhaps a few hundredths of a second. The solid line in Figure 8.1 is a graph showing what this force might look like as a function of time.

We began this chapter by setting a goal of being able to work with forces that varied, though we derived the impulse–momentum equation by considering a constant applied force only. However, the key to working with varying forces may be seen in Figure 8.1. When the force varies, its total impulse is found by considering it as a nearly infinite number of tiny impulses of duration δt. Each small impulse starts with an initial v_{i} and changes it to a final v_{f}, which then becomes the v_{i} for the next small impulse, and so on. The δt widths are chosen to

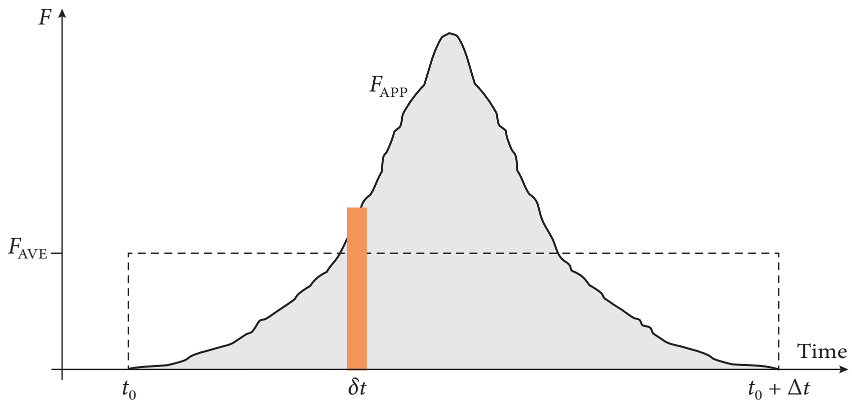


Figure 8.1 An actual force applied to some object (the solid line) over a time Δt versus the average force applied during the same time (dashed line). The red-shaded narrow rectangle is the impulse for a tiny segment of the time, δt.

be short enough that the force is nearly constant during that time, so that the tiny increment of impulse during the time δt is the instantaneous value of F_{APP} times δt. The total impulse is found by adding up all the tiny incremental impulses. When total impulse is defined in this way, it will always satisfy Equation 8.2. Note also that the impulse produced over the tiny δt shown in Figure 8.1 is the height in newtons of the narrow red-shaded rectangle in the figure times its width in seconds. So this small impulse is proportional to the area of the red-shaded rectangle in the figure, meaning that the total impulse between t_0 and $t_0 + \Delta t$ will be proportional to the gray-shaded area between the $F = 0$ axis and the F_{APP} curve.

Figure 8.1 also shows a dashed-line rectangle whose height is the average force F_{AVE} and whose width is Δt. The dashed-line rectangle has the same area as the area under the F_{APP} curve. This means that F_{AVE} is the force which, if it acted over the entire time Δt, would have the same total impulse (1500 N · s) as the actual force F_{APP}. For instance, if the bumper car collision in Example 8.1 lasted for $\Delta t = 0.05$ seconds, we would have $F_{\text{AVE}} \times 0.05\ \text{s} = 1500\ \text{N}\cdot\text{s}$. This would mean that the average force by the wall on the bumper car was $F_{\text{AVE}} = 1500\ \text{N}\cdot\text{s}/0.05\ \text{s} = 30{,}000\ \text{N}$. This clearly doesn't tell us everything about the collision. For instance, if the graph is correct, the peak force acting on the bumper car was two to three times greater than this average force. However, in many cases, an estimate of the average force will be all we need.

EXAMPLE 8.2 Impulse on a Hockey Puck

A 1-kg practice hockey puck is sliding east at 5 m/s on a frozen pond. Marc deflects the puck with his stick so that the force by his stick on the puck is directed due north and hit with such a careful touch that the average force he applies is 100 N over a time of 0.05 s. What is the final velocity (speed and direction) of the puck?

ANSWER Given the average force acting on the puck and the duration of the interaction, we can find the impulse. This determines how the puck's momentum changes during the deflection. We know enough to find the initial momentum, so adding the change will give us the final momentum.

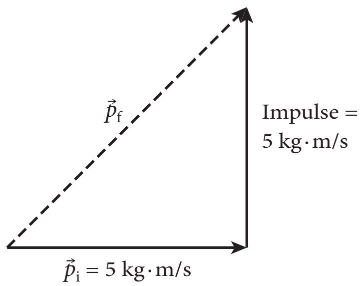
$$\vec{p}_{\mathrm{f}} = \vec{p}_{\mathrm{i}} + \overrightarrow{\text{Impulse}}$$

$$\vec{p}_{\mathrm{f}} = m\vec{v}_{\mathrm{i}} + \vec{F}_{\mathrm{AVE}}\Delta t$$

The initial momentum is (1 kg)(5 m/s), or 5 kg·m/s, to the east. We can calculate the impulse to be (100 N)(0.05 s), or 5 N·s (or kg·m/s), to the north. If we write these vectors in component form, with the *x*-component toward the east, we have

$$\vec{p}_{\mathrm{f}} = (5\ \mathrm{kg\cdot m/s},\ 5\ \mathrm{kg\cdot m/s}).$$

A vector diagram is shown at right. We see the direction of the final momentum is northeast. Using the Pythagorean theorem, we find the magnitude of $\vec{p}_{\mathrm{f}}$ to be 7.1 kg·m/s. We have only to find the corresponding velocity. Since the puck has a mass 1 kg, its velocity will be $v = p_{\mathrm{f}}/m$ = 7.1 m/s to the northeast.



8.2 Collisions

In the last section we found that the idea of impulse provided a practical way to approach forces that vary in time, like those that occur during a collision between two bodies. Momentum then arose as the quantity that appeared in the impulse–momentum equation, an alternative version of Newton's second law. However, the real utility of the concept of momentum comes when we look at the total momentum of an isolated system of bodies that interact or collide with one another.

An *isolated system* is a set of bodies that have no interaction with the outside world. We recall from Chapter 7 that the *total energy* of an isolated system is strictly conserved. But we also saw that this wasn't the last word on the subject, because *mechanical energy* may or may not be conserved, depending on the types of forces acting between the bodies of the system. When there are non-conservative forces involved, mechanical energy can be converted into other forms. Mechanical energy is evident in the speed and position of the bodies in the system, but, if mechanical energy turns into thermal energy, it is no longer visible in the motion alone. This is the case for energy. However, as we shall soon see, the *total momentum* of a system has no such back-door exit strategies available. It cannot hide inside a body like thermal energy does. It is always apparent in the motion. This is what makes the idea of momentum so useful in dealing with isolated systems.

A "collision" is always divided into three phases. First, there is a phase when the bodies have no significant interaction with each other, having constant initial momenta. Second, there is a short time when the bodies interact via internal forces that produce equal and opposite impulses on each body, changing their momenta. Third, there is a final phase where the bodies are again no longer interacting with each other, and the final momenta are again constant.

To see how this all works, let us consider a system consisting of two objects, A and B, that are about to collide with each other, as in Figure 8.2a. We assume that there are no forces acting on either of the bodies from anything outside the system, or, at least, that any external forces may be neglected because they are small compared to the collision forces. We write the initial momenta of the two bodies as $\vec{p}_{\mathrm{A}}^{\mathrm{i}}$ and $\vec{p}_{\mathrm{B}}^{\mathrm{i}}$, where the subscripts indicate the object and the superscripts indicate that this is the initial state. The objects then collide with one another for a short time Δt, after which they move off with new momenta $\vec{p}_{\mathrm{A}}^{\mathrm{f}}$ and $\vec{p}_{\mathrm{B}}^{\mathrm{f}}$ (Figure 8.2b).

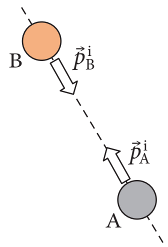


Figure 8.2a The initial momenta.

Each object's change in momentum is described by the impulse–momentum theorem for that object. For object A, this gives

$$\vec{p}_A^{\,f} = \vec{p}_A^{\,i} + \vec{F}_{B,A}\Delta t, \tag{8.3}$$

where $\vec{F}_{B,A}$ is the average force by B on A during the collision. Similarly, for object B we get

$$\vec{p}_B^{\,f} = \vec{p}_B^{\,i} + \vec{F}_{A,B}\Delta t.$$

Now we need to realize that $\vec{F}_{A,B}$ and $\vec{F}_{B,A}$ are Newton's third-law companion forces, so that $\vec{F}_{A,B} = -\vec{F}_{B,A}$. We can therefore rewrite the last equation as

$$\vec{p}_B^{\,f} = \vec{p}_B^{\,i} - \vec{F}_{B,A}\Delta t. \tag{8.4}$$

If we now add Equations 8.3 and 8.4 together, remembering that the time intervals are the same in both equations, the terms containing the forces will cancel, leaving

$$\vec{p}_A^{\,i} + \vec{p}_B^{\,i} = \vec{p}_A^{\,f} + \vec{p}_B^{\,f}. \tag{8.5}$$

On each side of Equation 8.5 we have a vector sum, and we certainly know how to add vectors by now. In each case, we are vector-adding the momentum vector for body A to the momentum vector for body B. The result of this operation is a new vector, which we will obviously want to call the total momentum of the system,

$$\vec{p}_{\mathrm{TOT}} = \vec{p}_A + \vec{p}_B.$$

So, on the left-hand side of Equation 8.5, we have the initial total momentum of the system, $\vec{p}^{\,i}_{\mathrm{TOT}}$, and, on the right-hand side, we have the final total momentum, $\vec{p}^{\,f}_{\mathrm{TOT}}$. Then you see what Equation 8.5 means. As long as no external forces act on either body in the system, there is nothing the two bodies can do to each other—no weird non-conservative force, no ugly irregular time-varying force—that will change the total momentum of the system. *As long as a system of bodies is isolated from the outside world*, the total momentum of the system is drop-dead conserved. It is an absolutely reliable fact of the motion. Other things may change. Kinetic energy may turn into thermal energy, atomic nuclei may decay and become different nuclei with different masses, but *the total momentum of the system just never ever changes.*

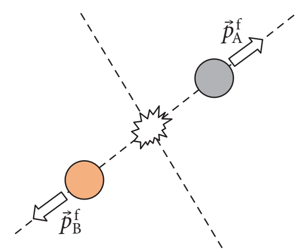
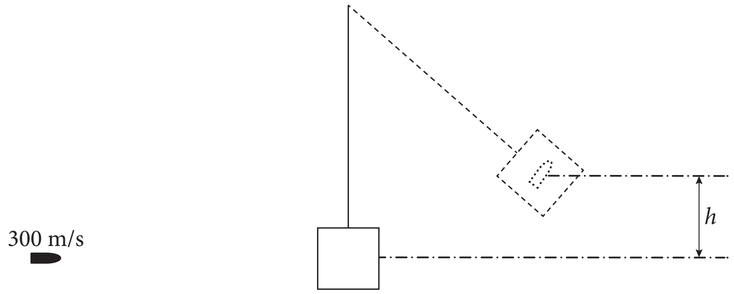


Figure 8.2b The final momenta.

EXAMPLE 8.3 The Ballistic Pendulum

A 10-g bullet is fired horizontally at 300 m/s into a 2-kg block of wood suspended on the end of a long string fastened to the ceiling. What is the speed of the block with the bullet inside immediately after the bullet embeds in the block? What is the maximum gain in height that the block+bullet achieves as it swings?



ANSWER This is really two problems in one. The purpose of the first part is to make clear that, even in a complicated setup like this one, a collision is a collision, and momentum is conserved. The second part is what happens after the collision. Here, there are external forces, gravity and rope tension, making it an energy problem like many we saw in the previous chapter. This example also demonstrates that to apply these concepts we must learn when it will be useful to apply energy concepts and when momentum concepts will be most productive. We will also give some helps for addressing this question in the next section.

First, we treat the collision using the concept of momentum conservation. We take as our system the block + bullet because these are the two things that collide. The collision itself only happens in one dimension (the two-dimensional motion doesn't start until after the collision is over), so this is a one-dimensional momentum conservation problem. We take the direction of the initial momentum of the bullet to be positive. The initial momentum of the system is found by adding up the momenta of the constituents:

$$p^{\mathrm{i}}_{\mathrm{TOT}} = m_{\mathrm{bullet}} v^{\mathrm{i}}_{\mathrm{bullet}} + m_{\mathrm{block}} v^{\mathrm{i}}_{\mathrm{block}} = (0.01\ \mathrm{kg})(300\ \mathrm{m/s}) + (2.0\ \mathrm{kg})(0\ \mathrm{m/s}) = +3\ \mathrm{kg\cdot m/s}.$$

After the collision, the total momentum must still be $+3\ \mathrm{kg\cdot m/s}$ although the momenta of the individual parts will have changed. If the bullet collides with a block, there are an infinite number of combinations of bullet and block speeds that would add up to $+3\ \mathrm{kg\cdot m/s}$. However, in this case we have an additional constraint. We know that the bullet ends up embedded in the block, so the two move with the same final velocity, which we call v^{f}. Conservation of momentum then requires

$$p^{\mathrm{f}}_{\mathrm{TOT}} = m_{\mathrm{bullet}} v^{\mathrm{f}} + m_{\mathrm{block}} v^{\mathrm{f}} = (m_{\mathrm{bullet}} + m_{\mathrm{block}}) v^{\mathrm{f}} = (2.01\ \mathrm{kg}) v^{\mathrm{f}} = 3\ \mathrm{kg\cdot m/s}.$$

This gives

$$v^{\mathrm{f}} = 1.49\ \mathrm{m/s}.$$

So, the block+bullet system moves at 1.49 m/s immediately following the collision.

In the second part, we recognize that the block+bullet system now swings on the end of a rope that does no work on the system, so energy is now conserved. As the system swings, it converts kinetic energy into gravitational potential energy. At the maximum height, all of the energy is in the potential energy bucket.

Just after the collision and before the block+bullet starts to rise, the energy is

$$E_{\mathrm{initial}} = \mathrm{KE}_{\mathrm{initial}} + \mathrm{PE}_{\mathrm{initial}} = \frac{1}{2}(2.01\ \mathrm{kg})(1.49\ \mathrm{m/s})^2 + 0 = 2.23\ \mathrm{J}.$$

The fact that this total does not change allows us to find the maximum height of the swinging pendulum. We write

$$E_{\mathrm{final}} = 2.23\ \mathrm{J} = \mathrm{KE}_{\mathrm{final}} + \mathrm{PE}_{\mathrm{final}} = 0 + (m_{\mathrm{bullet}} + m_{\mathrm{block}})gh$$

$$\text{or} \quad h = \frac{E_{\mathrm{final}}}{(m_{\mathrm{bullet}} + m_{\mathrm{block}})g} = \frac{2.23\ \mathrm{J}}{(2.01\ \mathrm{kg})(10\ \mathrm{N/kg})} = 0.11\ \mathrm{m}.$$

Although it wasn't required to solve the problem in Example 8.3, we can find out how much kinetic energy was lost during this collision, if we want. Before striking the block, the bullet's kinetic energy was $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(0.01)(300)^2 = 450$ J, and the block was at rest. So, immediately before the collision, there were 450 J of kinetic energy in the system. Immediately after the collision there were only 2.23 J of kinetic energy, as we calculated above. Thus, about 99.5% of the initial kinetic energy was lost to other forms of energy (mostly thermal). Clearly, if we had assumed that kinetic energy would be conserved during the collision we would have badly overestimated the speed of the block after the collision (and, of course, violated the principle of the conservation of momentum). So we hope the moral of this story is clear: *You cannot count on kinetic energy being conserved in a collision, but you can always count on momentum being conserved in a collision.*

8.3 The Three Flavors of Collisions

Imagine two 1500-kg cars, each driving at 10 m/s, about to hit head-on (see Figure 8.3). The momenta of the two cars have the same magnitude but they are pointing in opposite directions. This makes the initial total momentum of the system exactly zero.

Figure 8.3 Two cars approaching each other initially.

Although the tires of the cars are in contact with the pavement when they hit, the magnitude of the force between the two cars is so much greater than friction with the pavement or other forces from the environment, that the system of the two cars is effectively an isolated system. This means that the total momentum will always be the same. In particular, it must still be zero after the collision. Figure 8.4 shows three particular cases among the infinite possibilities for the final velocities. All of these cases satisfy the conservation of a zero total momentum.

In Figure 8.4a, the cars have the same speeds they had coming into the collision, so the total kinetic energy after the collision is the same as it was before the collision. Kinetic

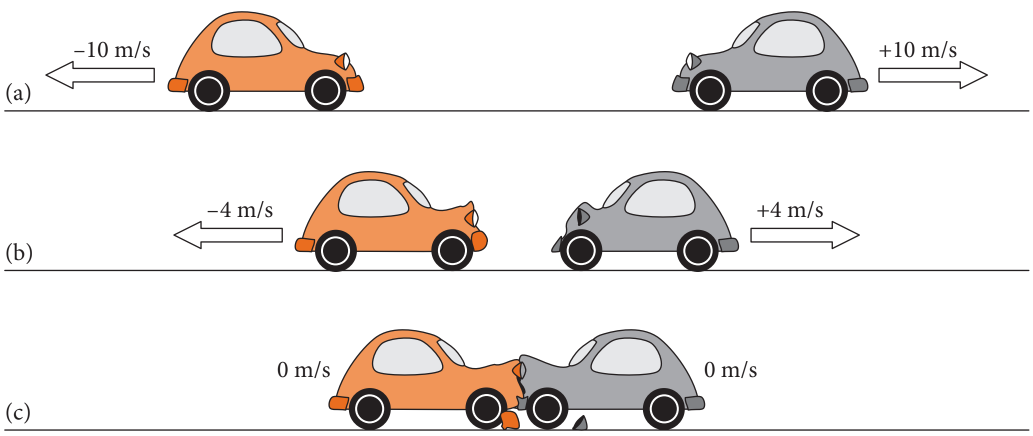


Figure 8.4 Three possible outcomes of the collision, all preserving zero total momentum.

energy has been conserved. This could happen only if the forces between the two cars were all conservative forces, like perfectly springy bumpers that could turn the cars around without any loss of energy. Collisions that conserve kinetic energy like this are called *elastic*, or *totally elastic*. Most collisions that we encounter do not meet this standard, since most collisions lose some energy (for example, sound waves carry energy, so, if you hear the collision, some of the kinetic energy has been converted to sound waves). Generally, the only way you can count on kinetic energy being conserved in a problem is if the problem tells you that the collision is elastic.

In Figure 8.4b, we see that there has been significant loss of kinetic energy during the collision. At least some of the forces that deformed the fronts of the cars were non-conservative, and a lot of the initial kinetic energy has been transformed to other forms of energy (mostly to thermal energy). We use the term *inelastic collision* to describe collisions that lose some kinetic energy.

Finally, let us consider Figure 8.4c. Here, the two cars have locked bumpers and stuck together, coming to a complete stop. The final kinetic energy is zero. The system has lost as much kinetic energy as it could (in this case, all of it). Collisions that lose the maximum possible kinetic energy are called *maximally inelastic*.[1]

Let us consider the maximally inelastic case a bit more. In the two-car collision of Figure 8.4, the total conserved momentum was always zero, so it was possible for both cars to end up at rest. All of the initial kinetic energy was lost. However, think what could have happened if the red car had initially been moving to the right at 12 m/s, instead of 10 m/s. Then the initial total momentum would have been (1500 kg)(12 m/s) – (1500 kg)(10 m/s) = +3000 kg · m/s, to the right. But, the collision must conserve momentum, since the two cars constitute an isolated system. This means that it would not be possible to end up with both cars at rest. The equal and opposite impulses acting on the two cars during the collision cannot simultaneously bring both cars to a stop. If the magnitude of the impulse is exactly the right amount to stop the gray car, it will not be the amount needed to stop the red car, and vice versa. So, if the initial momentum is non-zero, then the final momentum will be non-zero, and we must always end up with something in the system that is moving.

Thus, the way to tell if a collision is maximally inelastic is not to look for a case where everything comes to a stop, but to note that a maximally inelastic collision is one in which all the bodies involved in the collision end up stuck together, all moving in the same direction (the direction of the initial momentum vector) at the same speed. This is the situation that satisfies a non-zero momentum requirement with the least excess kinetic energy. The bullet+block collision we studied in Example 8.3 is an example of a maximally inelastic collision. We know that it is maximally inelastic because the two objects became locked together. And, as we saw, that system lost about 99.5% of its initial kinetic energy during the collision.

1. Many textbooks refer to these collisions as "perfectly inelastic" or "completely inelastic." We find these terms misleading, since they seem to suggest that all the initial kinetic energy is lost in the collision. Since this is not generally the case, we prefer the term "maximally inelastic."

EXAMPLE 8.4 A One-Dimensional Collision

A 10-kg block is sliding to the right across a frictionless floor at 4 m/s. A 5-kg block is traveling left at 2 m/s such that it hits the other block head-on. ("Head-on" is our way of promising that nothing in the collision will change the one-dimensional nature of this problem.) After the collision, the 10-kg block is observed moving to the right at 1 m/s. Is this collision elastic, inelastic, or maximally inelastic?

ANSWER Before we can identify the type of collision, we need to know the final velocity of the 5-kg block. For instance, if we found that it was also moving at 1 m/s to the right, we would know the blocks were stuck together (maximally inelastic). To find the 5-kg block's final velocity we use the fact that momentum is conserved. We have enough information to find the initial momentum of the system:

$$\vec{p}^{\,\mathrm{i}}_{\mathrm{TOT}} = (10\ \mathrm{kg})(+4\ \mathrm{m/s}) + (5\ \mathrm{kg})(-2\ \mathrm{m/s}) = +30\ \mathrm{kg \cdot m/s},$$

where we have chosen positive to mean to the right. After the collision, we know the momentum of the 10-kg block. It is

$$\vec{p}^{\,\mathrm{f}}_{10\ \mathrm{kg}} = (10\ \mathrm{kg})(+1\ \mathrm{m/s}) = +10\ \mathrm{kg \cdot m/s}.$$

Since the total must still equal +30 kg · m/s, the final momentum of the 5-kg block must be +20 kg · m/s. Thus, the 5-kg block is moving to the right with a speed of 4 m/s after the collision. The two final velocities are not the same, so we know that the collision was not maximally inelastic. To determine whether the collision is elastic or inelastic we must compare the kinetic energies before and after the collision. The two numbers are

$$\mathrm{KE}_{\mathrm{initial}} = \frac{1}{2}(10\ \mathrm{kg})(4\ \mathrm{m/s})^2 + \frac{1}{2}(5\ \mathrm{kg})(2\ \mathrm{m/s})^2 = 80\ \mathrm{J} + 10\ \mathrm{J} = 90\ \mathrm{J}$$

$$\mathrm{KE}_{\mathrm{final}} = \frac{1}{2}(10\ \mathrm{kg})(1\ \mathrm{m/s})^2 + \frac{1}{2}(5\ \mathrm{kg})(4\ \mathrm{m/s})^2 = 5\ \mathrm{J} + 40\ \mathrm{J} = 45\ \mathrm{J}.$$

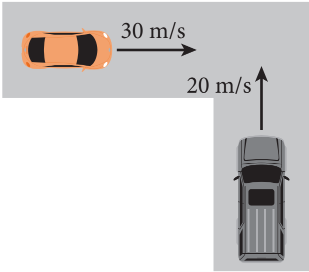
Because the kinetic energy of the system is reduced during the collision, and because the two blocks do not become locked together, we conclude that the collision is inelastic, but not maximally inelastic. Note that knowing that the kinetic energy is reduced is enough to determine that the collision is inelastic, but it is not enough to distinguish a maximally inelastic collision. For that, we must also know if the objects end up with the same final velocity.

8.4 Collisions in Two Dimensions

Example 8.2 emphasized that the impulse–momentum equation is a vector equation. The quantities involved, forces and velocities, have directions that must be considered, in addition to the magnitudes. Yet, so far, we have dealt only with collisions that happen along a straight line—one-dimensional collisions. Although we did have to remember that momentum is a vector in solving these problems, the vector part of the problem was easily handled by picking one direction to be positive and the other negative. Now, we will expand our horizons a bit. Our real world has three dimensions, of course, but the mathematics required in a full 3-dimensional analysis of a collision is much harder than for two, and it doesn't really add very much to the physics. So let us just look at a two-dimensional problem and see how we must now be very explicit in our treatment of the vector nature of momentum.

Based on the philosophy that a long gory example is sometimes the best way to put everything together, we choose to present . . . a long gory example.

EXAMPLE 8.5 The Red Car and the Gray Truck



A red 1000-kg car is driving due east at 30 m/s. At the same time, a gray 2000-kg truck is driving due north at 20 m/s. The two collide in an intersection and lock bumpers so that they stick together. After the collision, they both have their brakes locked, so they are skidding together. Assume that the coefficient of kinetic friction between the tires and the road for both vehicles is exactly 0.9.

a) What is the speed of the pair immediately following the collision?
b) In what direction does the pair move following the collision?
c) What impulse does the car apply to the truck during the collision?
d) Is the collision elastic, inelastic, or maximally inelastic?
e) How far does the pair slide before coming to rest?

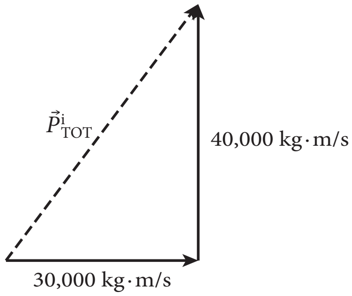
ANSWER This multi-part problem reviews most of the concepts from this chapter as well as providing a little more practice using the work–energy theorem (part e).

Part a The first two parts of this problem are a direct application of the principle of momentum conservation in a collision. We begin by finding the initial momentum of the system. The *magnitudes* of the two momenta are

$$p^{\mathrm{i}}_{\mathrm{car}} = (1000\ \mathrm{kg})(30\ \mathrm{m/s}) = 30{,}000\ \mathrm{kg \cdot m/s}$$
$$p^{\mathrm{i}}_{\mathrm{truck}} = (2000\ \mathrm{kg})(20\ \mathrm{m/s}) = 40{,}000\ \mathrm{kg \cdot m/s}.$$

If we define a coordinate system with the x-axis to the east and the y-axis to the north, the total initial momentum can be written in component notation as

$$\vec{p}^{\,\mathrm{i}}_{\mathrm{TOT}} = (30000{,}40000)\ \mathrm{kg \cdot m/s}.$$



We show the components of the initial total momentum as the solid lines in the diagram at right. The vector triangle is a 3-4-5 triangle, so the magnitude of the initial momentum (and of the final momentum, for that matter) is 50,000 kg·m/s. Note also that the direction of the total momentum is 53° north of east, and that this is true even though no object is actually moving in that direction. This vector will be conserved all through the collision. As long as the system is isolated, nothing can ever happen to change the total momentum vector. After the collision, the vehicles end up locked together, creating a single object of mass 3000 kg moving at some velocity $\vec{v}^{\,\mathrm{f}}$. Since the mass of the two locked cars times their speed must give us $mv^{\mathrm{f}} = 50{,}000\ \mathrm{kg \cdot m/s}$, the speed of the cars immediately after the collision must be $v^{\mathrm{f}} = (50000\ \mathrm{kg \cdot m/s})/(3000\ \mathrm{kg})$, or 16.7 m/s.

Part b Immediately after the collision, there is only one object, so that object must be moving in the direction of the system's momentum. It will therefore move in the same direction as the initial total momentum, or 53° north of east.

Part c The impulse that the car applies to the truck must, according to the impulse–momentum theorem, be equal to the change in the truck's momentum. We know that the truck's momentum before the collision is 40,000 kg · m/s north. To find the momentum of the truck after the collision, we use its mass and speed to get the magnitude of the momentum as

$$p^{\mathrm{f}}_{\mathrm{truck}} = (2000\ \mathrm{kg})(16.7\ \mathrm{m/s}) = 33{,}400\ \mathrm{kg\cdot m/s};$$

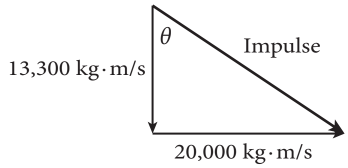
and the direction will be 53° north of east. To find the impulse, we need to find the change in momentum, the final minus the initial. It is convenient to first divide the truck's final momentum into components:

$$p^{\mathrm{f}}_{\mathrm{truck},x} = 33{,}400\cos(53°) = 33{,}400(0.6) = 20{,}000\ \mathrm{kg\cdot m/s};$$
$$p^{\mathrm{f}}_{\mathrm{truck},y} = 33{,}400\sin(53°) = 33{,}400(0.8) = 26{,}700\ \mathrm{kg\cdot m/s}.$$

We then find that

$$\mathrm{Impulse}_x = 20{,}000\ \mathrm{kg\cdot m/s} - 0 = 20{,}000\ \mathrm{kg\cdot m/s};$$
$$\mathrm{Impulse}_y = 26{,}700\ \mathrm{kg\cdot m/s} - 40{,}000\ \mathrm{kg\cdot m/s} = -13{,}300\ \mathrm{kg\cdot m/s}.$$

These impulse vector components are diagrammed at right. We use our usual vector methods to find that the magnitude of the impulse on the truck iz 24,000 kg · m/s and that the angle of the impulse is $\theta = 56°$, south of east. The impulse that the truck applied to the car would have the same magnitude, but it would point in the opposite direction. You might want to verify this by redoing the above calculation for the car.



Part d In this collision, the objects are locked together and have the same final velocity. Thus we know that the collision is maximally inelastic.

Part e Probably the easiest way to solve this problem is to use the work–energy theorem. If we consider the motion of the car and truck from the time immediately after the collision until they have come to rest, the mechanical energy of the system is decreased through a process we understand—kinetic friction during the skid. So we can solve this part of the problem by finding the mechanical energy of the system immediately after the collision and asking how far it must slide to get rid of this energy. The only mechanical energy of interest here is the kinetic energy.

$$\mathrm{KE}_{\text{after collision}} = \frac{1}{2}m_{\mathrm{total}}v^2 = \frac{1}{2}(3000\ \mathrm{kg})(16.7\ \mathrm{m/s})^2 = 418{,}000\ \mathrm{J}.$$

This energy is reduced to zero during the skid because of the work done by the kinetic frictional force. We can find the magnitude of this force from

$$f_{\mathrm{road,tires}} = \mu_{\mathrm{K}} N_{\mathrm{road,tires}} = \mu_{\mathrm{K}} m_{\mathrm{total}} g = (0.9)(3000\ \mathrm{kg})(10\ \mathrm{m/s^2}) = 27{,}000\ \mathrm{N}.$$

The work done by this force is the magnitude of the force multiplied by the distance that the vehicles slide. So, using d to represent the length of the skid, we can write

$$W = f_{\mathrm{road,tires}}\, d = \Delta E$$

$$d = \frac{\Delta E}{f_{\mathrm{road,tires}}} = \frac{418{,}000\ \mathrm{J}}{27{,}000\ \mathrm{N}} = 15.5\ \mathrm{m}.$$

Though complicated, this example integrates a number of important ideas from this chapter. It is worth reviewing several times to be sure that you see the "big picture" and don't get lost in the technical details. By the way, you might be interested to know that this kind of thing is exactly what police departments do in reconstructing an accident. They measure the lengths of incoming skid marks to determine how much kinetic energy was lost before the collision; they assume that momentum was conserved during the collision; and they measure the lengths of the outgoing skid marks to determine the kinetic energy lost after the collision. This way, they are able to determine each car's speed going into the accident and see if anyone was exceeding the speed limit.

So, let us sum up. Here is the procedure for handling problems involving collisions. First, make sure that you have included in the system all bodies that will bump into each other. Second, you might as well just go ahead and write down Equation 8.5, because you know you are going to have to use it eventually. Pick out the masses of whatever bodies you have, resolve all initial and final velocities into components, and then write down as many versions of Equation 8.5 as you have components in the problem. Some velocities or masses or angles may be unknown, and you will have to keep them as letters to be solved for. Note that the number of dimensions in the problem will tell you how many equations you will have and, therefore, how many unknowns will be determined. For motion that you know is confined to one dimension, Equation 8.5 will give a single equation that can determine one unknown. If the motion is going to be two-dimensional, there will be two independent equations that come out of Equation 8.5, and you will be able to determine two unknowns by using the single principle of conservation of momentum.

Finally, let us be clear on the limitation of the momentum method for solving collision problems. As we mentioned back in Section 1.1, the purpose of the laws of physics is to be able to correctly predict the future state of some system, knowing the details of an initial state. If we know the initial velocities of two objects in a collision, and the details of the forces that act during the collision, Newton's laws are sufficient to exactly predict the final velocities of both objects after the collision. In a two-dimensional collision of two bodies, that means predicting both final speeds and both final directions—four quantities. The two components of the momentum conservation equation in two dimensions provide only two independent equations, which is not enough to completely determine the final state of things. What is missing in, say, a two-car collision is the detail of the forces involved in the crunching of bumpers and grilles. While such an analysis is possible in principle, no one will do it in practice. Fortunately, the law of conservation of momentum gives a lot of information without having to go into that level of detail at all.

In our next section, however, we discuss one of the few situations where the conservation laws are sufficient to completely specify the final state after a collision. This is the case of elastic collisions in one dimension.

8.5 Elastic Collisions in One Dimension

In this section, we add no new physics, but simply consider how to use the conservation of kinetic energy in problems where we know that the collision is elastic. Just a warning—the algebra we need to do for the solution here is a little tougher than our average example. But take it slowly and you'll be OK.

EXAMPLE 8.6 An Elementary Particle Collision in One Dimension

Two elementary particles, of identical mass, travel in colliding beams from laboratory accelerators. One, call it particle A, is moving to the right at a speed 4×10^6 m/s and the other, particle B, is moving to the left at a speed of 3×10^6 m/s. The beams are exactly aligned, so that the collision is head-on and the two particles rebound back along the directions they came. What will be the final velocities of the two particles?

ANSWER First, let us point out that one of the few times that we can assume that a collision is elastic without being told is when the colliding objects are elementary particles. This is because they have no internal structure where they can hide any energy in another form. Second, although there are no details about the kind of force that acts between the two particles, the fact that the entire collision is one-dimensional already limits a lot of the details of the interaction. No non-symmetric interactions will kick these particles away from the line of their incoming directions. Since the collision is elastic, we know that kinetic energy is conserved, in addition to the usual momentum conservation we can count on in all collisions in isolated systems. This means that there will be *two* independent equations relating the initial and final velocities of the particles. And *this* means that we will have enough information from conservation laws alone to determine the two unknown final velocities.

We are not given the mass of the two identical particles, so let us designate this with a letter m and hope that it will not matter in the final solution. Conservation of momentum in one dimension lets us write

$$mv_A^i + mv_B^i = mv_A^f + mv_B^f,$$

and energy conservation requires

$$\frac{1}{2}m(v_A^i)^2 + \frac{1}{2}m(v_B^i)^2 = \frac{1}{2}m(v_A^f)^2 + \frac{1}{2}m(v_B^f)^2.$$

Sure enough, m cancels out of both of these equations. If we take velocities to the right as positive, we have $v_A^i = 4 \times 10^6$ m/s and $v_B^i = -3 \times 10^6$ m/s, so the two equations become

$$v_A^f + v_B^f = 4 \times 10^6 - 3 \times 10^6 = 1 \times 10^6$$

$$(v_A^f)^2 + (v_B^f)^2 = (-3 \times 10^6)^2 + (4 \times 10^6)^2 = 25 \times 10^{12}.$$

Please note that, after having carefully checked that all numbers are in meters and seconds, we have dropped the units in these two equations.

If you are a little rusty on how to solve two equations for two unknowns, you might want to check out the algebra review we have back in Appendix A. We can solve the first equation for v_B^f, giving $v_B^f = (1 \times 10^6) - v_A^f$, and substitute this in the second equation to get

$$(v_A^f)^2 + [1 \times 10^6 - v_A^f]^2 = 25 \times 10^{12} \quad \text{or} \quad (v_A^f)^2 = 25 \times 10^{12} - [1 \times 10^6 - v_A^f]^2.$$

Squaring the term in the square brackets turns this into

$$(v_A^f)^2 = 25 \times 10^{12} - [1 \times 10^{12} - (2 \times 10^6)v_A^f + (v_A^f)^2]$$

or

$$(v_A^f)^2 = 24 \times 10^{12} + (2 \times 10^6)v_A^f - (v_A^f)^2.$$

If we bring everything to the left side of the equation, we get

$$2(v_A^f)^2 - (2 \times 10^6)v_A^f - 24 \times 10^{12} = 0.$$

We can divide by 2, giving a simple quadratic equation:

$$(v_A^f)^2 - (10^6)v_A^f - 12 \times 10^{12} = 0.$$

There are always two solutions to a quadratic equation. We use the quadratic formula (see Appendix A.4) to get

$$v_A^f = \begin{cases} 4 \times 10^6 \\ -3 \times 10^6. \end{cases}$$

First, let's be clear why there are two solutions. The initial velocity of particle A was 4×10^6 m/s to the right and the first solution has the final velocity of particle A as 4×10^6 m/s. This would be the right answer if particle A never hit particle B. But remember that the equations only tell us what final velocities conserve momentum and kinetic energy. They have no way of knowing that a collision occurred. But *we* know that there was a collision, so we ignore the first solution and take the second. If v_A^f is -3×10^6 m/s, then the final velocity of particle B can be found from

$$v_B^f = 1 \times 10^6 - v_A^f = 1 \times 10^6 - (-3 \times 10^6) = 4 \times 10^6.$$

The two particles have exchanged velocity. Particle A started out at 4×10^6 m/s and ended up going -3×10^6 m/s, while particle B started with -3×10^6 m/s and ended up going 4×10^6 m/s.

Note that the answer did not depend on the nature of the forces between the two particles. They could have been two protons, repelling each other electrically (which we will learn about in Chapter 16), or they could have been two neutrons, interacting only at close distance via the strong nuclear force (see Chapter 25). The details of the motion during the time the forces are acting would be very different in those two cases, but, once the forces have had their effects, the conservation laws are enough to correctly predict the velocities.

8.6 Summary

In this chapter you should have learned the following:

- You should be able to calculate the impulse acting on an object by noting the change in its momentum, using Equation 8.2. You should also be able to calculate impulse by use of Equation 8.1 when the force is constant.
- You should be able to carefully define the elements of an "isolated system" in which no forces intrude from the outside world or in which outside forces may be neglected compared to the large forces involved, say, in a collision.
- You should be able to write down the equations of momentum conservation in one or two dimensions, and to use these to solve for unknowns in the initial or final state of the system.
- You should be able to recognize where, in a multi-step problem, there is a momentum-conserving collision that may be used to connect properties of the system before the collision to those after, and then to recognize where momentum conservation may no longer apply. See Examples 8.3 and 8.5.

CHAPTER FORMULAS

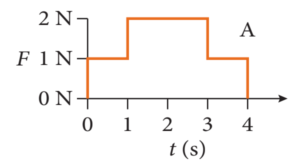
Impulse: $\overrightarrow{\text{Impulse}} = \vec{F}_{\text{APP}}\Delta t$

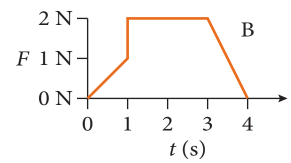
Momentum: $\vec{p} = m\vec{v}$

Impulse–momentum: $\overrightarrow{\text{Impulse}} = \vec{p}_{\text{f}} - \vec{p}_{\text{i}}$

Momentum conservation: $\vec{p}^{\,\text{i}}_{\text{A}} + \vec{p}^{\,\text{i}}_{\text{B}} = \vec{p}^{\,\text{f}}_{\text{A}} + \vec{p}^{\,\text{f}}_{\text{B}}$

PROBLEMS





* 1. What is the impulse of a constant force of 300 N acting for 0.01 s?

* 2. What is the impulse provided by a force whose magnitude as a function of time is given by graph A at right?

** 3. What is the impulse provided by a force whose magnitude as a function of time is given by graph B at right?

* 4. In a collision of a car with a wall, the wall acts over a time 0.15 s and provides an impulse of 4000 N · s. What is the average force of the wall on the car?

* 5. What is the momentum of a 60-kg woman running north at a speed of 4 m/s?

* 6. What impulse is required to stop a 160 g pool ball moving at 2 m/s?

** 7. A softball with mass 200 g is thrown horizontally directly toward a brick wall. It is traveling at 15 m/s just before hitting the wall and rebounds from the wall at 10 m/s, still traveling horizontally. The ball is in contact with the wall for 0.02 s and the ball is compressed a maximum of 7 mm.

a) What is the magnitude of the ball's change in momentum from just before to just after striking the wall?

b) What is the magnitude of the average force of the wall on the ball?

** 8. A molecule of mass 3×10^{-26} kg is moving toward a wall at a speed of 3×10^5 m/s. It bounces back in exactly the opposite direction going at the same 3×10^5 m/s.

a) What was the magnitude of the change in momentum of the molecule?

b) What was the magnitude of the impulse applied to the molecule?

c) If the molecule interacted with the wall for 1 μs (10^{-6} s), what was the average force of the wall on the molecule?

d) What impulse is applied by the molecule to the wall?

** 9. Object A is more massive than object B, but they have the same momentum. You stop each of them with the same retarding force. Answer and explain the following:

a) Which one will stop in the shorter time, or will the times be the same?

b) Which one will stop in the shorter distance, or will the distances be the same?

** 10. A 1500-kg car and a 4000-kg truck have the same momentum. The car's kinetic energy is 6×10^5 J. What is the kinetic energy of the truck?

** 11. A 4-kg block slides without friction along a tabletop at 10 m/s. A constant force of 20 N is applied until the block is going in the opposite direction at 10 m/s. Over what period of time was the force applied?

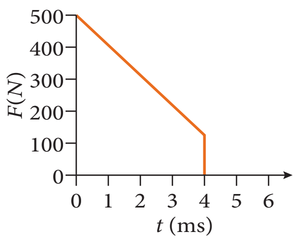
** 12. A 100-gram beach ball is dropped from a height of 1.8 m. After it bounces off the floor, it rises to a height of only 0.8 m.

a) How fast was it going just before it hit the floor and how fast was it going just after it bounced?

b) What was the total impulse of the floor on the ball?

c) How much energy was lost in the bounce?

d) If the ball was in contact with the floor for 0.05 s, what was the average force of the floor on the ball?

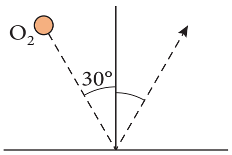


** 13. The force on a bullet in a rifle is given by the graph at left. The force starts at 500 N, falls off linearly to 100 N at the end of the barrel, and then drops quickly to zero. The mass of the bullet is 5 g. What is the muzzle velocity of the rifle?

** 14. A hockey puck of mass 0.160 kg is traveling north at a speed of 40 m/s when it is hit by a hockey stick that provides an impulse of 6.4 N·s toward the east.

a) What is the initial momentum of the hockey puck, before it hits the stick?

b) What is the final momentum (magnitude and direction) of the puck?



*** 15. A single oxygen molecule of mass 5.3×10^{-26} kg hits the wall of an oxygen tank moving at a speed of 480 m/s and at an angle 30° from the normal to the tank wall, as shown at left. It rebounds elastically at an equal angle and speed. What total impulse (magnitude and direction) is imparted to the molecule by the wall?

* 16. A 5-kg rifle, not held tightly against a shooter's shoulder, fires a 20-gram bullet at a speed of 450 m/s. What is the recoil speed of the rifle? (Note: This will hurt.)

* 17. An empty boxcar traveling at 10 m/s approaches a string of 4 identical boxcars sitting empty and stationary on the track. The moving boxcar collides and links with the stationary cars and the 5 move off together along the track. What is the final speed of the 5 cars immediately after the collision?

* 18. An ice fisherman slides a 0.40-kg can of bait along the ice toward a friend at a speed of 3 m/s. It hits his friend's 3.2-kg tackle box that is initially resting on the ice. Right after the collision, the bait can ends up stopped on the ice. What is the speed of the tackle box right after the collision?

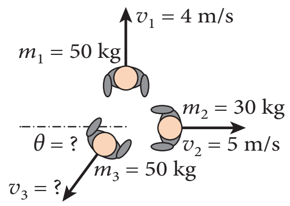
* 19. A 100-kg hockey player wants to find the mass of his figure-skating girlfriend. He asks her to push him away when they are both initially at rest on the ice and he observes that he then moves backwards from a mark on the ice at 2 m/s while she moves away from the same mark at 3 m/s. What is the figure skater's mass?

** 20. A 4-kg block slides to the right at 12 m/s on a frictionless table and collides head-on with a 6-kg block that is moving to the left, also at 12 m/s. Right after the collision, the small block is seen to be moving to the left at 3 m/s. Find the final velocity of the large block.

** 21. A neutron of mass 1.67×10^{-27} kg moves at a speed 4.00×10^5 m/s and strikes a nucleus of helium-3 (mass 5.01×10^{-27} kg) that is initially at rest. The neutron is absorbed by the nucleus and forms a nucleus of helium-4 with mass 6.64×10^{-27} kg. What is the final speed of the helium-4 nucleus?

** 22. Three children are standing together on a totally frictionless pond. At the count of three, they all push against each other in an attempt to escape to the edge of the pond. Child 1

(m_1 = 50 kg) ends up moving north at 4 m/s. Child 2 (m_2 = 30 kg) ends up moving east at 5 m/s. Child 3 has mass m_3 = 50 kg and ends up moving at an angle θ south of west, as shown.

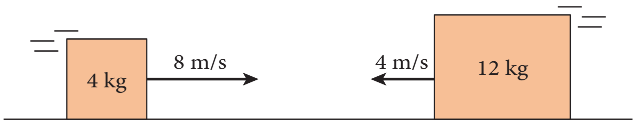


a) Is the *magnitude* of the impulse experienced by Child 1 as they push off greater than, equal to, or less than the *magnitude* of the impulse experienced by Child 2 as they push off? Explain.

b) Find the final velocity of Child 3 (magnitude and direction).

** 23. A 2-kg mortar shell in a fireworks display rises from the ground to a height at which it is moving directly upward at 10 m/s. At this instant, the shell explodes and splits into two pieces. A 0.5-kg piece heads off northward and exactly horizontally at a speed of 20 m/s. What is the velocity (magnitude and direction) of the second piece? What is the change in kinetic energy during the explosion?

** 24. A small block (m_S = 4 kg) and a large block (m_L = 12 kg) experience a head-on collision on a frictionless tabletop. Before the collision, the small block was moving 8 m/s to the right and the large block was moving at 4 m/s to the left, as shown.

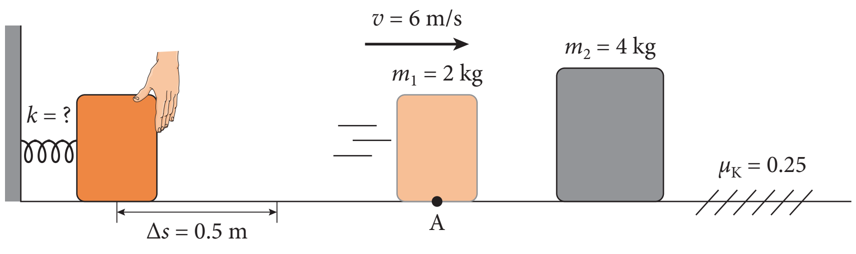


a) If the small block bounces back from the collision with a velocity 3 m/s to the left, find the final velocity (magnitude and direction) of the large block.

b) Suppose instead that the two blocks stick together when they collide. Now what will the final velocity of the two stuck-together blocks be?

c) How many joules of kinetic energy would be lost in the collision in part (b)?

** 25. A small block of mass m_1 = 2 kg is pushed by hand to compress a spring a distance Δs = 50 cm. The hand then releases the block and the spring launches it across a frictionless surface toward a large block of mass m_2 = 4 kg that initially is not moving. The small block is moving at velocity v = 6 m/s just before it strikes the large block. After the collision, the large block is observed to be moving at a velocity of 4 m/s to the right. The large block then slides over a long, rough section of the track having a coefficient of kinetic friction μ_K = 0.25 exactly, and is eventually brought to a stop after a time interval Δt.



a) Find the spring constant for the spring.

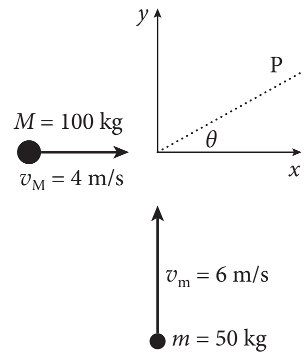
b) Find the velocity (magnitude and direction) of the small block just after the collision.

c) What is the system's kinetic energy just before the collision?

d) What is the system's kinetic energy just after the collision?

e) Is this collision maximally inelastic, inelastic, or elastic? Explain your reasoning.

f) For how many seconds will the large block slide across the rough section of track before it comes to a halt? (Assume g = 10 m/s^2.)



** 26. Two football players are running along perpendicular paths. The masses and velocities are given in the figure at left. The small one tackles the larger one at the point represented by the origin of the coordinate system and the two stick together, sliding in the mud along the line marked P. The coefficient of friction between the players and the mud is 0.20. You may assume that the force exerted by the ground on the players' cleats is negligible compared to the huge force they exert on each other. Find:

a) The total momentum of the system before the collision (magnitude *and* direction).

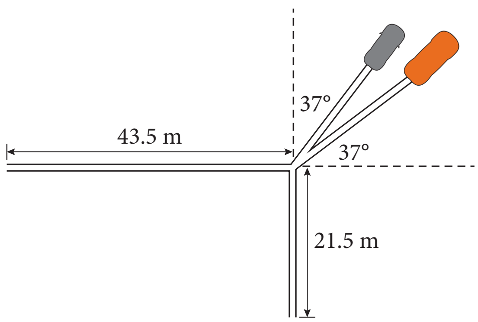
b) The final momentum of the system just after the collision (magnitude and direction).

c) The final velocity of the players (magnitude and direction).

d) The kinetic energy of the system just after the collision.

e) The distance the players slide before coming to rest.

** 27. A desperate Little League outfielder, seeing a fly ball going over his head, throws his mitt at it. Miraculously, the ball hits the mitt and sticks in it. If the ball is going exactly horizontally at 28 m/s at the instant it hits the mitt, and if the mitt is moving straight upward at a speed of 6 m/s when the ball hits it, find the momentum vector (magnitude and direction) of the mitt with the ball in it. The mass of a baseball is 0.143 kg and the mass of the mitt is 0.6 kg.

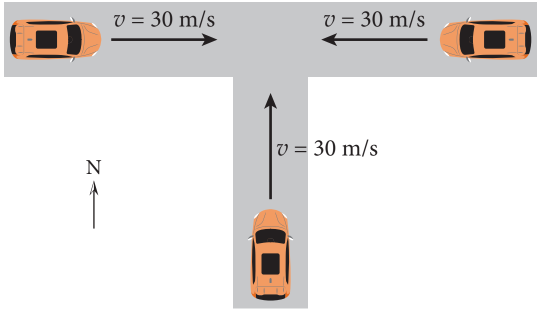


*** 28. A police investigator arrives at an accident scene where he finds that a gray 1000-kg Honda Civic has collided with a red 2000-kg SUV. From the skid marks, he is able to determine that the Honda was initially traveling due east when the driver hit the brakes, while the SUV was going due north. After the collision, the Honda skidded 37° east of north (what other angle could it be?) for a distance of 3.5 m, while the SUV skidded 37° north of east for a distance of 4.0 m. The pre-collision skid marks of the Honda are 43.5 m long and the pre-collision skid marks of the SUV are 21.5 m long, as shown. The coefficient of friction of Honda tires with the pavement is 0.7, while that of the SUV tires is 0.8. Help the investigator work out the initial speeds of the two vehicles before they hit their brakes.

a) From the lengths of the skid marks after the collision, find the speeds of each vehicle just after the collision.

b) Use conservation of momentum during the collision to find the speed of each vehicle just before the collision. [Hint: Remember that momentum is a vector.]

c) From the lengths of the skid marks before the collision, determine the initial speeds of the two vehicles before they hit their brakes.



*** 29. Three identical Volvos, each of mass 2000 kg, enter an intersection, one heading west, one heading east, and one heading north. Each of them is initially traveling at 30 m/s. The three collide and lock bumpers, becoming one single mangled Volvo-thing with a dozen wheels.

a) Find the total momentum (magnitude and direction) of the three-car system just before the collision.

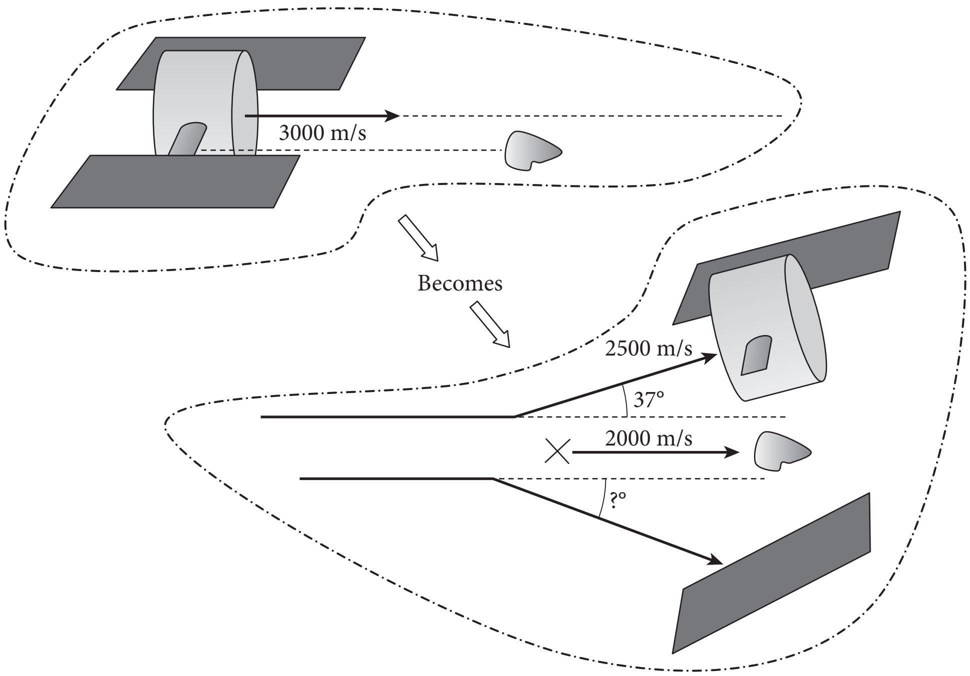
b) What is the total momentum (magnitude and direction) of the three-car system just after the collision? Explain your reasoning.

c) Find the velocity (magnitude and direction) of the mangled Volvo-thing just after the collision.

d) Find the kinetic energy of the Volvo-thing just after the collision.

e) If the coefficient of kinetic friction between the Volvo-thing and the road is $\mu_K = 0.2$, find the distance the Volvos will slide before coming to rest.

*** **30.** A spacecraft of mass 200 kg (including the solar panels) is moving at a speed of 3000 m/s toward an initially stationary 10-kg meteoroid. The meteoroid strikes the spacecraft in the middle of the strut that holds one of the two 20-kg solar arrays, severing that solar array from the spacecraft. The meteoroid is kicked directly forward, along the initial direction of the spacecraft, at a speed of 2000 m/s. The collision knocks the spacecraft, minus the one of its solar arrays, away from its initial path. It ends up moving at a speed of 2500 m/s at an angle of 37° from the initial direction. Find the velocity, magnitude, and direction of the severed solar array.

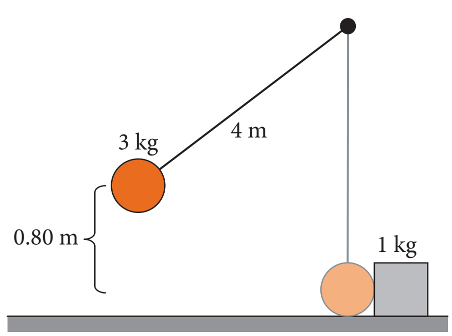


** **31.** A pendulum consists of a ball of mass 3.0 kg on a string of length 4.0 m. The ball is pulled back until its center has risen to a height of 0.80 m. The ball is released such that it strikes a 1.0-kg block, as shown. The speed of the block after the collision is measured to be 6.0 m/s.

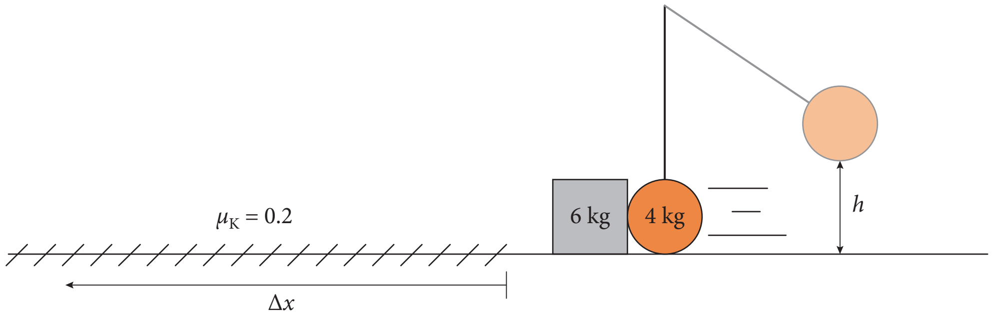
a) What is the speed of the ball just before the collision?

b) What is the speed of the ball immediately after the collision?

c) What flavor of collision is this (elastic, inelastic, or maximally inelastic)? Justify your answer.



** **32.** A pendulum bob of mass 4 kg is released from height h and swings down to collide with a stationary block of mass 6 kg. Right before the collision the bob is moving to the left at speed 5 m/s. Just after the collision, the block is traveling to the left at speed 3 m/s. The block then slides over a very long, rough surface with a coefficient of kinetic friction that is exactly $\mu_K = 0.2$, bringing the block to a stop at a point Δx past the beginning of the rough section.



a) Find the height H from which the bob was released.

b) Find the velocity (magnitude and direction) of the bob right after the collision.

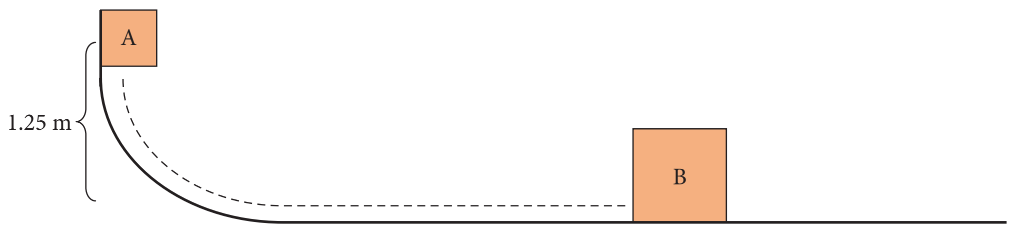
c) What is the bob–block system's kinetic energy just before the collision?

d) What is the bob–block system's kinetic energy just after the collision?

e) Is this collision maximally inelastic, inelastic, or elastic? Explain.

f) How far will the block slide across the rough section of track before it comes to a halt?

** 33. Block A, with mass m = 2 kg, slides down a frictionless track so that its center of mass drops 1.25 m, as shown. Block A then collides with block B, which has mass M = 4 kg. After the collision, block B is observed moving to the right with speed 2 m/s.



a) How fast is block A moving when it first reaches the flat portion of the ramp?

b) Find the velocity (magnitude and direction) of block A after the collision.

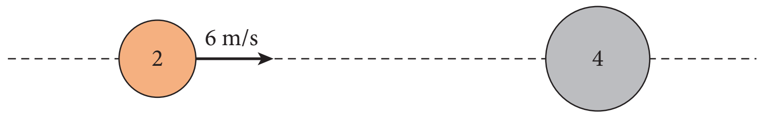
c) What is the system's kinetic energy just before the collision?

d) What is the system's kinetic energy just after the collision?

e) Is the collision inelastic, elastic, or maximally inelastic? Explain.

*** 34[f]. Consider the case where a hard-hit cue ball of mass m is sliding across a pool table at speed v and strikes an initially stationary 3-ball head-on in an elastic collision. The 3-ball has an identical mass m. Find a formula for the final speeds of both balls in terms of the initial speed v.

*** 35. A ball of mass 2 kg moving at a velocity 6 m/s to the right strikes a 4-kg ball that is initially stationary. Assume that the collision is head-on and elastic, and find the final velocities of the two balls.



*** 36. A 7-kg bowling ball hits a 1½-kg pin head-on. After the collision, the bowling ball is still moving down the bowling alley. Its speed is 2 m/s. Assume that the collision is *elastic*. Find the initial speed of the ball before it hit the pin and the speed of the pin just after the collision with the ball.