Chapter 13 Figures From

MATHEMATICAL METHODS for Scientists and Engineers

Donald A. McQuarrie

For the Novice Acrobat User or the Forgetful

When you opened this file you should have seen a slightly modified cover of the book *Mathematical Methods for Scientists and Engineers* by Donald A. McQuarrie, a menu bar at the top, some index markers at the left hand margin, and a scroll bar at the right margin.

Select the **View** menu item in the top menu and be sure **Fit in Window** and **Single Page** are selected. Select the **Window** menu item and be sure **Bookmarks** and **Thumbnails** ARE NOT selected.

You can and probably should make the top menu bar disappear by pressing the function key F9. Pressing this key (F9) again just toggles the menu bar back on. You may see another tool bar that is controlled by function key F8. Press function key F8 until the tool bar disappears.

In the upper right hand corner margin of the window containing this text you should see a few small boxes. DO NOT move your mouse to the box on the extreme right and click in it; your window will disappear! Move your mouse to the second box from the right and click (or left click); the window containing this text should enlarge to fill the screen. Clicking again in this box will shrink the window; clicking again will return the display to full screen.

The prefered means of navigation to any desired figure is controlled by the scroll bar in the column at the extreme right of the screen image. Move your mouse over the scroll bar slider, click, and hold the mouse button down. A small window will appear with the text "README (2 of 41)". Continuing to hold down the mouse button and dragging the button down will change the text in the small window to something like "13.4 (6 of 41)". Releasing the mouse button at this point moves you to Figure 13.4 of Chapter 13. The (6 of 41) indicates that Figure 13.4 resides on page 6 of the 41 pages of this document.

ANIMATIONS

Figures 13.38 and 13.39 have associated animations that require QuickTime™ for display. The animations are named Anim13_38.mov and Anim13_39.mov; they must be independently downloaded from the server.

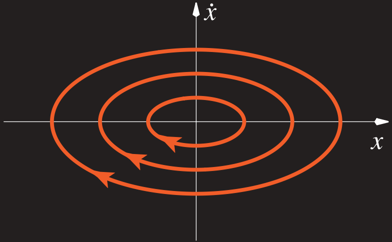

Figure 13.1
The phase portrait of a simple harmonic oscillator. The arrows indicate the forward direction of time.

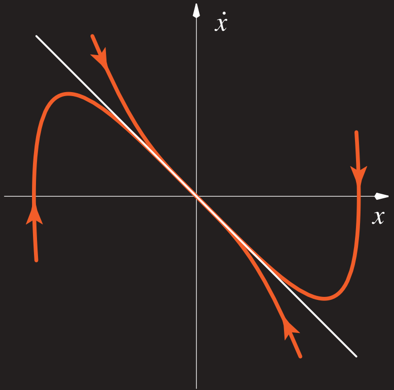

Figure 13.2
The phase portrait of an overdamped harmonic oscillator.

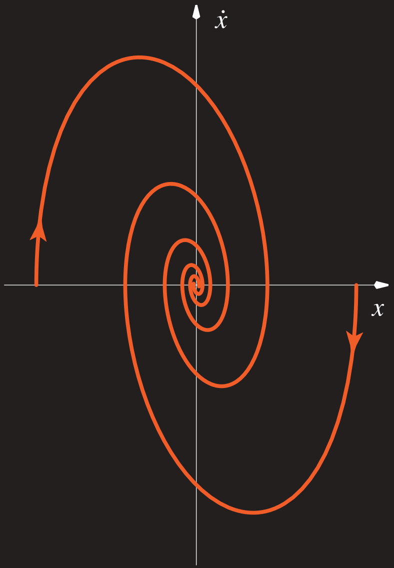

Figure 13.3
The phase portrait of an underdamped harmonic oscillator.

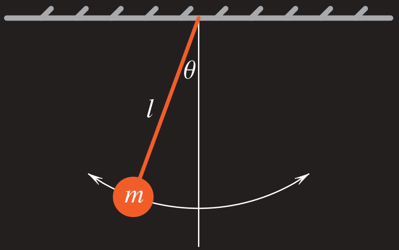

Figure 13.4
An illustration of a pendulum consisting of mass m supported by a mass-less rigid rod of length l swinging in a fixed plane.

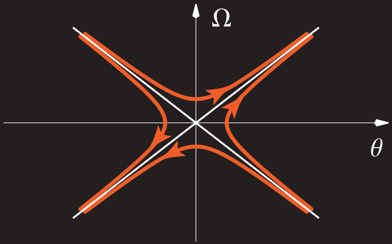

Figure 13.5
The trajectories of the pendulum described by Equation 13 about the point $\Omega = 0$, $\theta = \pi$ for various initial conditions with $\omega = 2$.

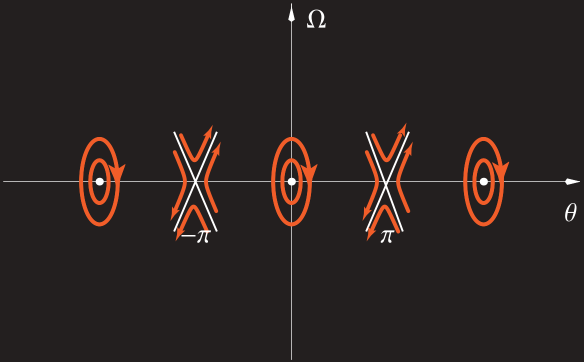

Figure 13.6
A partial phase portrait of the pendulum described by Equation 13, showing the trajectories around the centers at $(0, \pm 2n\pi)$ and the saddle points at $(0, \pm(2n+1)\pi)$ for $n = 0, 1, 2, \ldots\ldots$

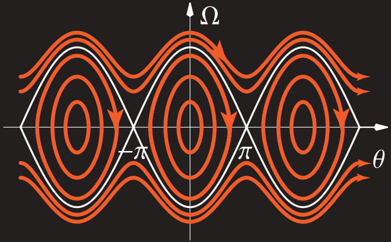

Figure 13.7
The phase portrait of the pendulum described by Equation 13.

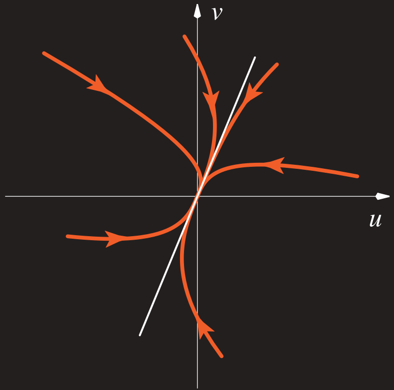

Figure 13.8
The trajectories of Equation 8 for various valuse of c_1 and c_2. The critical point is an improper node.

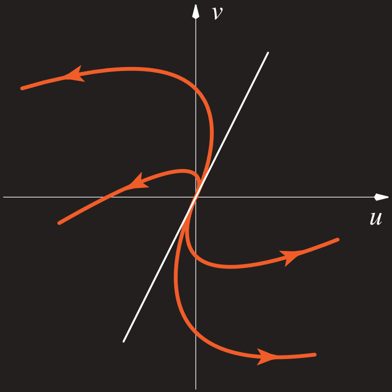

Figure 13.9
The trajectories of Example 3 for various values of c_1 and c_2. The critical point is an improper node.

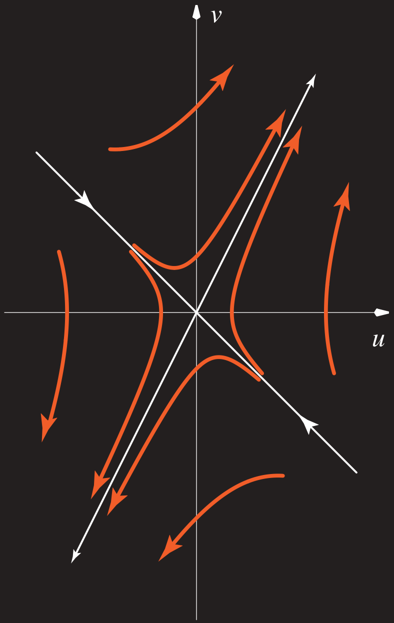

Figure 13.10
The trajectories of Equation 10 for various values of c_1 and c_2. The critical point is a saddle point.

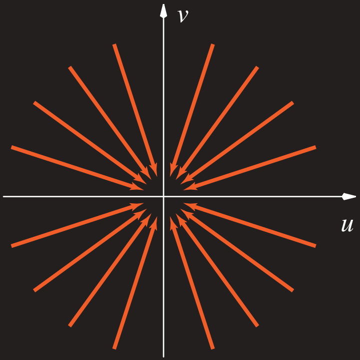

Figure 13.11
An illustration of a proper node with the trajectories approaching the critical point.

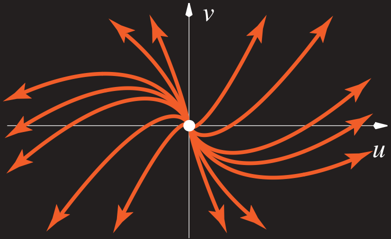

Figure 13.12
The trajectories given by Equation 15 for various values of c_1 and c_2. The critical point is an improper node.

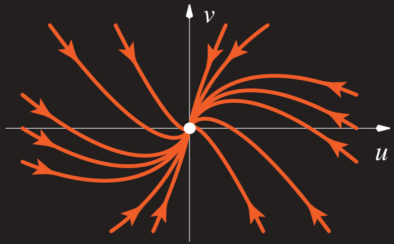

Figure 13.13
The phase portrait for Example 4. The critical point is an improper node.

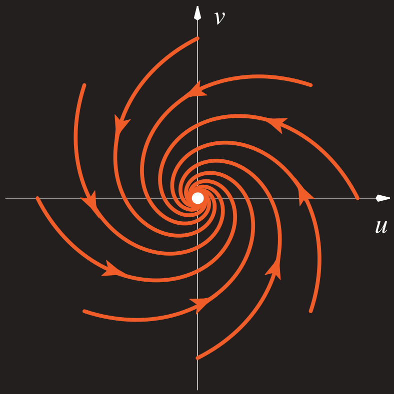

Figure 13.14
An illustration of the spiral point described by Equation 18.

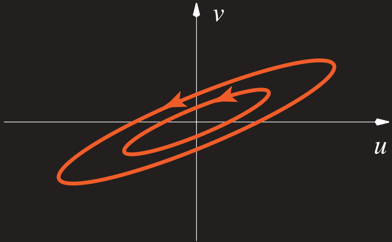

Figure 13.15
Two trajectories of the system described in Example 5. The critical point is a center.

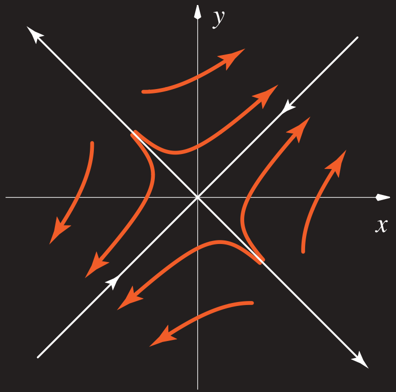

Figure 13.16
The trajectories around the saddle point associated with the system in Example 2, illustrating that a saddle point is unstable.

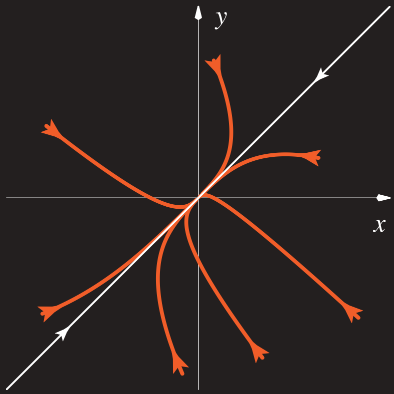

Figure 13.17
The trajectories described by Example 3, showing that the critical point (0, 0) is an asymptotically stable improper node. Note that all the trajectories approach the origin tangentially to the straight line $y = x$ (black) as t increases.

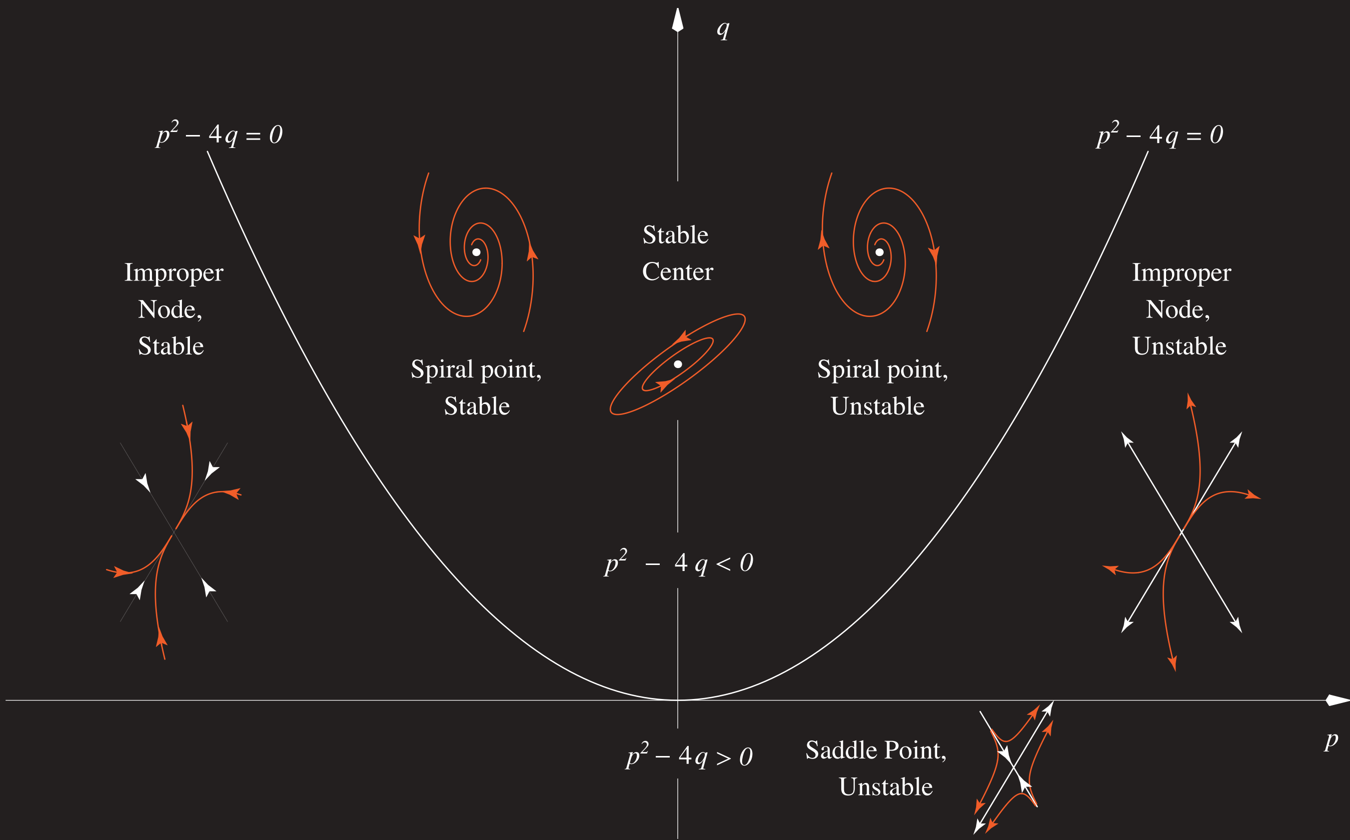

Figure 13.18
A diagram showing the various types of nodes and their stabilities as a function of $q = ad - bc$ and $p = a + d$. There is a center all along the vertical line $p = 0$ for $q > 0$ and the parabola $\Delta = p^2 - 4q = 0$ separates different types of critical points.

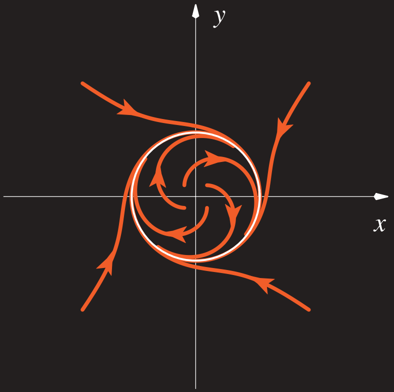

Figure 13.19
Equation 6 plotted against t for values of $r_0 > 1$ and $r_0 < 1$. In both cases, the trajectories spiral toward the unit circle.

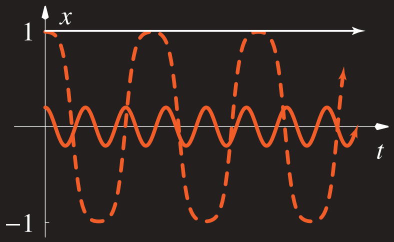

Figure 13.20

A numerical solution to the equation $\ddot{x} + x - x^3 = 0$ for the initial conditions (a) $x(0) = 0.20$ and $\dot{x}(0) = 0$ (solid color); (b) $x(0) = 0.99$ and $\dot{x}(0) = 0$ (dashed color); (c) $x(0) = 1$ and $\dot{x}(0) = 0$ (white). Note that curve c is simply a horizontal line.

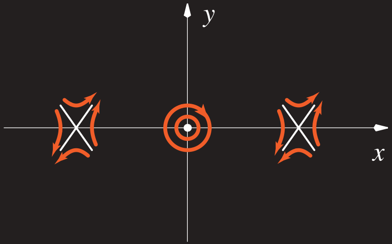

Figure 13.21
The phase portrait of Equations 2 about its three critical points at (0, 0) and (±1, 0).

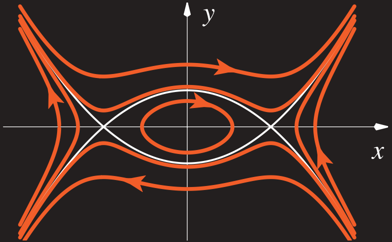

Figure 13.22
The family of closed trajectories given by Equation 6 for various values of C. The separatrix, which is shown in white, is given by Equation 5.

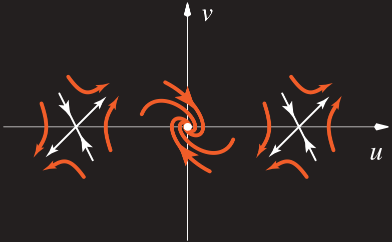

Figure 13.23
The phase portrait for the system described in Example 1 about its three critical points.

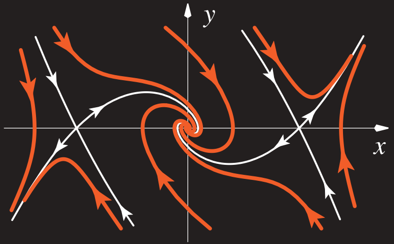

Figure 13.24
The complete phase protrait of the system described in Example 1.

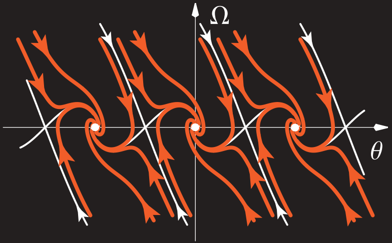

Figure 13.25
The phase portrait of an underdamped pendulum of arbitrary amplitude described by Equations 8.

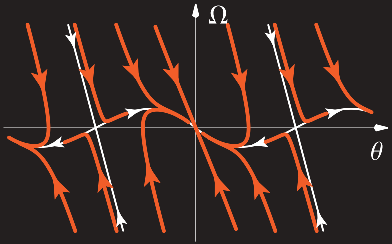

Figure 13.26
The phase portrait of an overdamped pendulum of arbitrary amplitude described by Equations 8.

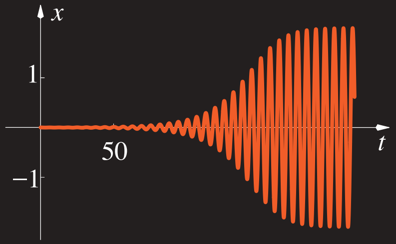

Figure 13.27
The displacement of a van der Pol oscillator plotted against time for $x(0) = 0.0010$ and $\dot{x}(0) = 0$.

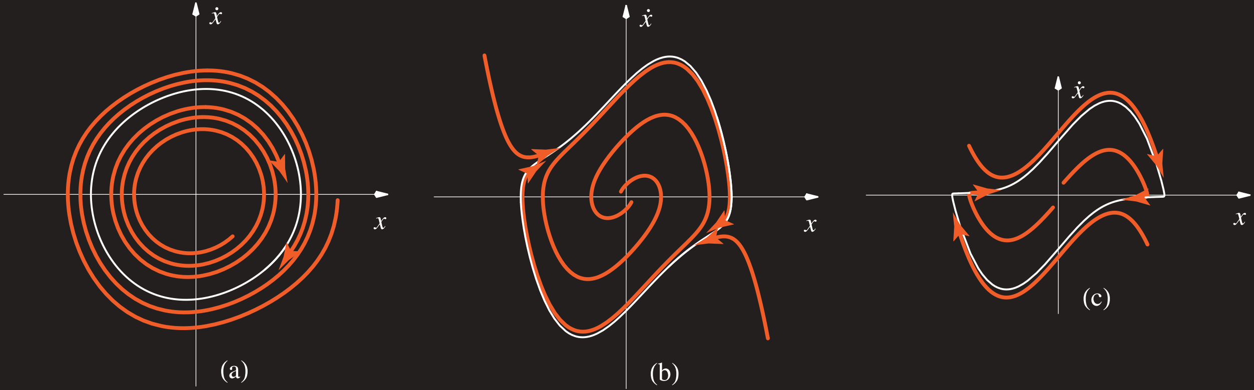

Figure 13.28
An illustration of the limit cycle of the van der Pol equation obtained numerically for $\epsilon = 0.10$ (a), 1.0 (b), and 5.0 (c).

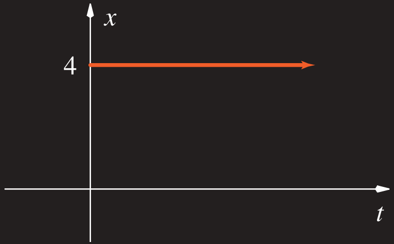

Figure 13.29
The numerical solution to $\ddot{x} + x - \frac{1}{4}x^2 = 0$ for the initial conditions $x(0) = 4$, $\dot{x}(0) = 0$.

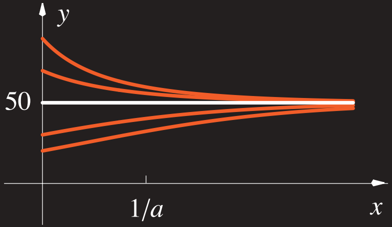

Figure 13.30
Equation 3 plotted against t for various values of P_0.

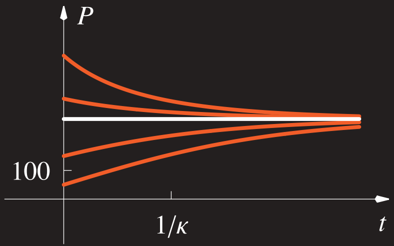

Figure 13.31
Equation 4 plotted against t for $a = 0.020$, $b = 0.00020$, and $I = 10$ for various values of P_0. The limiting value of P_0 is $[a + (a^2 + 4\,b\,I)^{1/2}]/2b = 279$.

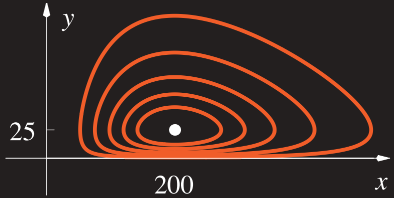

Figure 13.32
Equation 8 with $a = 1.0$, $b = 0.040$, $c = 4.0$, and $d = 0.020$ plotted in the phase plane.

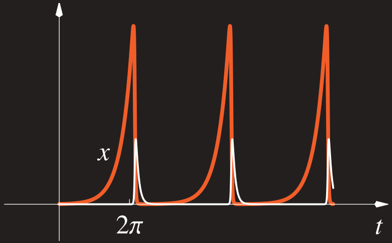

Figure 13.33
Plots of $x(t)$ (color) and $y(t)$ (white) from Equations 5 and 6 with $a = 1.0$, $b = 0.040$, $c = 4.0$, and $d = 0.020$ plotted against t.

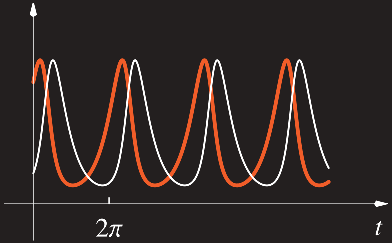

Figure 13.34
The numerical solution of Equations 5 and 6 for $x(t)$ (color) and $y(t)$ (white) for $a = b = c = d = 1$ and $x(0) = 2.0$ and $y(0) = 0.50$.

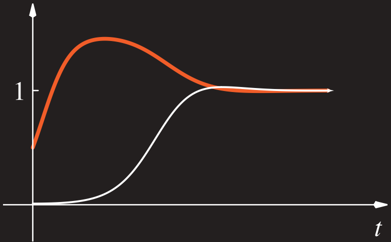

Figure 13.35
The numerical solution of Equations 10 for $x(t)$ (color) and $y(t)$ (white) for the initial conditions $x(0) = 1/2$ and $y(0) = 0.010$.

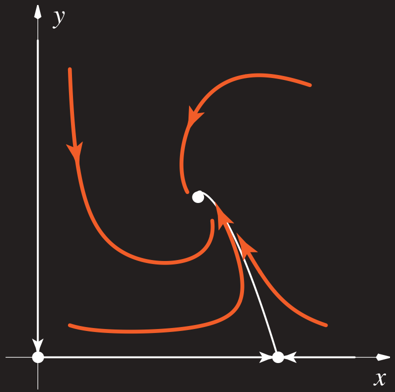

Figure 13.36
The phase portrait for the system described by Equations 10.

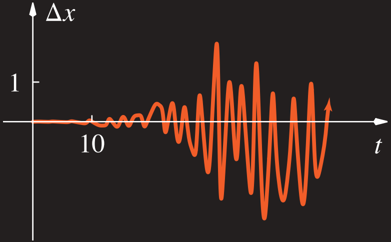

Figure 13.37
A plot of the difference between $x(t)$ calculated for the initial condidions $x(0) = 2.000$, $\dot{x}(0) = 0$ and $x(0) = 2.002$, $\dot{x}(0) = 0$, a 0.10% difference in $x(0)$.

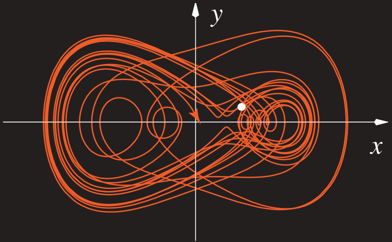

Figure 13.38
A parametric plot of y(t) against x(t) for Equations 12.

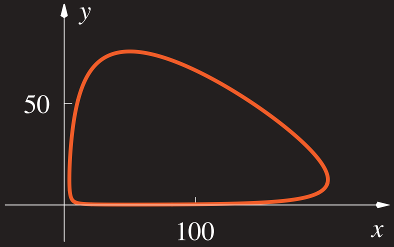

Figure 13.39
A parametric plot of the system described in Problem 9.